

MAT 334
Homework # 2
Due: Thursday, September 21st, 2017

Directions: Write careful solutions to each of the following problems on separate sheets of paper. (You may put more than one solution on the same sheet of paper, if you have enough room). Be sure to show all of your work. You are allowed to talk to your classmates about these problems. If you do receive help from a classmate, be sure to give them credit by noting their name on your solution. All solutions should be written in your own words, regardless of if you've received help. Partial credit is available. Each problem is worth five points.

1. Let $*$ be an associative binary operation on a set S and let $x \in S$. Let $C_S(x) = \{a \in S \mid a * x = x * a\}$. Prove that $C_S(x)$ is closed under $*$.
2. Determine whether the given map φ is an isomorphism between the first binary structure and the second binary structure. If φ is an isomorphism, prove it. If not, explain why.
 - (a) $(\mathbb{Z}, +)$ with $(\mathbb{Z}, +)$, where $\varphi(n) = 2n + 5$.
 - (b) $(M_2(\mathbb{R}), \cdot)$ with $(\mathbb{R}, \cdot)$, where $M_2(\mathbb{R})$ is the set of all 2×2 matrices with real entries and $\varphi(A)$ is the determinant of the matrix A .
 - (c) $(\mathbb{R}, \cdot)$ with $(\mathbb{R}, \cdot)$, where $\varphi(x) = x^5$.
 - (d) $(F, +)$ with $(F, +)$, where F is the set of functions $f : \mathbb{R} \rightarrow \mathbb{R}$ that have derivatives of all orders and $\varphi(f) = f'$, the derivative of f .
3. Suppose that if $\varphi : (S, *) \rightarrow (S', *')$ is an isomorphism. Prove that $\varphi^{-1} : (S', *') \rightarrow (S, *)$ is also an isomorphism.

Bonus Point: How many different commutative binary operations $*$: $S \times S \rightarrow S$ can be defined on S ?