MAT 334
Fall 2017
Homework 1
Due: Thursday, September 7

Directions: *Write careful solutions to each of the following problems on separate sheets of paper. You may put more than one solution on a page if you have room. Your solutions must be your own, but you are encouraged to discuss these problems with your classmates. In the event that you discuss problem(s) with your classmates, cite them by stating at the top of the page who you worked with/received help from. Each problem is worth 5 points.*

1. Let $A = \{a, b, c\}$ and let $B = \{\#, \$, \smiley\}$. Determine whether each of the following relations between A and B specify a function mapping A to B. If the relation is not a function, explain why. If the relation is a function, determine if the function is injective, surjective, both, or neither. Justify your solutions.

(a) $\{(a, \#), (b, \smiley), (c, \$)\}$
(b) $\{(a, \smiley), (b, \smiley), (c, \smiley)\}$
(c) $\{(a, \#), (a, \smiley), (a, \$)\}$
(d) $\{(a, \#), (b, \$), (c, \#)\}$
(e) $\{(b, \#), (b, \smiley), (c, \$)\}$

2. (a) Show that the closed interval $[2, 5]$ has the same cardinality as the closed interval $[3, 7]$ by constructing a bijection from the first interval to the second.

(b) Show that the open interval $(0, 1)$ has the same cardinality as $\mathbb{R}$.

3. How many partitions does a set with 4 elements have? How many partitions does a set with 5 elements have? (For a set with 5 elements, writing out each partition may prove to be a bit tedious. It may be easier to devise a way to describe the partitions and use your descriptions to give a total count.)

4. Recall the Binomial Theorem:

$$(x + y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$ is the number of ways we can choose i things from n things without replacement and where order does not matter. Use this theorem to prove that

$$|P(A)| = 2^n \quad \text{for any set } A \text{ such that} \quad |A| = n.$$