MAT 302
Homework # 6
Due: Thursday, March 30th, 2017

Directions: Write careful solutions to each of the following problems on separate sheets of paper. (You may put more than one solution on the same sheet of paper, if you have enough room). Be sure to show all of your work. You are allowed to talk to your classmates about these problems. If you do receive help from a classmate, be sure to give them credit by noting their name on your solution. All solutions should be written in your own words, regardless of if you've received help. Partial credit is available. Each problem is worth five points.

1. Evaluate the following iterated integral by converting to polar coordinates.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \sin\sqrt{x^2+y^2}\, dydx$$

2. Let $a > 0$ and consider the lamina corresponding to the plane region R (a square) with vertices $(0,0), (0,a), (a,0)$ and (a,a). Let k be a constant and determine the center of mass of the lamina for each of the following densities:

(a) $\rho(x,y) = k$

(b) $\rho(x,y) = kx$

(c) $\rho(x,y) = ky$

3. Translate the region R from exercise 3 five units to the right and repeat the calculations (a), (b), and (c). Then make a conjecture about the change in center of mass of a lamina with constant density when it is translated c units horizontally, for some constant c. Is your conjecture true when the lamina has variable density?

4. Find the center of mass of the lamina corresponding to the region in the plane bounded by the graphs of $y = \frac{1}{1+x^2}$, $y = 0$, $x = 1$, and $x = 1$. Assume that the density at the point (x,y) is given by $\rho(x,y) = k$ for some constant k.