MAT 302
Homework # 4
Due: Thursday, February 23rd, 2017

Directions: Write careful solutions to each of the following problems on separate sheets of paper. (You may put more than one solution on the same sheet of paper, if you have enough room). Be sure to show all of your work. You are allowed to talk to your classmates about these problems. If you do receive help from a classmate, be sure to give them credit by noting their name on your solution. All solutions should be written in your own words, regardless of if you've received help. Partial credit is available. Each problem is worth five points.

1. Let $f(x,y) = x^2 - xy + y^2$ and let $\vec{v} = \langle 5, 12 \rangle$. Find the directional derivative of f in the direction of $\vec{v}$ using

   (a) the definition of the directional derivative

   (b) the gradient of f.

2. Find the direction of maximum increase of the function $f(x,y) = x^3 \sin y - x \cos y$ at the point $(5, \frac{\pi}{3})$.

3. Show that the surfaces defined by the equations

   $$x^2 + y^2 + z^2 - 8x - 12y + 4z + 42 \quad \text{and} \quad x^2 + y^2 + 2z = 7$$

   are tangent to each other at the point $(2, 3, -3)$ by finding the tangent planes to each surface at this point, and showing that the planes are the same.

4. Consider the elliptic cone given by

   $$x^2 - y^2 + z^2 = 0.$$

   (a) Find an equation of the tangent plane at the point $(5, 13, -12)$.

   (b) Find symmetric equations of the normal line at the point $(5, 13, -12)$.