MAT 301
Homework # 2
Due: Friday, September 22nd, 2017

Directions: Write careful solutions to each of the following problems on separate sheets of paper. (You may put more than one solution on the same sheet of paper, if you have enough room). Be sure to show all of your work. You are allowed to talk to your classmates about these problems. If you do receive help from a classmate, be sure to give them credit by noting their name on your solution. All solutions should be written in your own words, regardless of if you've received help. Partial credit is available. Each problem is worth five points.

1. For each of the following, suppose that $0 < a_n \leq b_n$ for all $n \geq 1$. Determine whether the following statements are true or false. If false, give a counterexample to show that it is false.

   (a) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges.

   (b) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

   (c) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

   (d) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.

2. Determine whether each of the following series converges or diverges. When possible, give the sum of the convergent series.

   (a) $\sum_{n=1}^{\infty} 2 \left(\frac{3}{4}\right)^n$

   (b) $\sum_{n=0}^{\infty} \frac{2\sqrt{n}}{n^3+1}$

   (c) $\sum_{n=1}^{\infty} 3 \left(\sqrt{2}\right)^n$

   (d) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

   (e) $\sum_{n=1}^{\infty} \frac{2}{\sqrt[3]{n^2}}$

   (f) $\sum_{n=1}^{\infty} \frac{n^2+2n+1}{n^4-4n+2}$

   (g) $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{(2n-1)\pi}{2}\right)}{n}$

3. Let k be a positive integer. Use the integral test to determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} n^k e^{-n}$$