MAT 301

Homework # 1

Due: Friday, September 8th, 2017

Directions: Write careful solutions to each of the following problems on separate sheets of paper. (You may put more than one solution on the same sheet of paper, if you have enough room). Be sure to show all of your work. You are allowed to talk to your classmates about these problems. If you do receive help from a classmate, be sure to give them credit by noting their name on your solution. All solutions should be written in your own words, regardless of if you've received help. Partial credit is available. Each problem is worth five points.

1. In each case below, give an example of a sequence satisfying the given condition, or explain why no such sequence exists.

   (a) A sequence that converges to π.

   (b) A nonconstant, monotonically decreasing sequence that converges to π.

   (c) A monotonically decreasing sequence that diverges.

   (d) A bounded, monotonically decreasing sequence that diverges.

   (e) A sequence $\{a_n\}$ that converges, such that the series $\sum a_n$ also converges.

   (f) A sequence $\{a_n\}$ that converges to π, such that the series $\sum a_n$ also converges.

2. Consider the sequence $\{a_n\} = \left\{ \frac{2n!+1}{n!+2} \right\}$.

   (a) Find the first five terms of the sequence.

   (b) Based on part (a), do you think the sequence is monotonic?

   (c) Prove your claim from part (b).

   (d) Based on part (a), do you think the sequence is bounded?

   (e) Prove your claim from part (d).

3. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{2}{(n+1)(n+2)}$$

converges or diverges. If it converges, find its sum.

4. A ball is dropped from a height of 10 feet and begins bouncing. The height of each bounce is 65% of the height of the previous bounce. Find the total vertical distance traveled by the ball.

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