MAT 300
Spring 2019
Homework # 3
Due: Friday, 2/15/19

Directions: Neatly write solutions to each of the following problems on separate sheets of paper. (You can put multiple problems on one page). Staple this page to the front of your finished proofs. Minimal to no credit will be given for solutions without appropriate justification. Each problem is worth 5 points.

1. Verify that the subspace topology on a subset S of a topological space T is, in fact, a topology.

2. Let $f : \mathbb{R} \to \mathbb{Z}$ be the "floor" function:

$$f(x) = n \qquad \text{whenever} \qquad n \in \mathbb{Z} \text{ and } n \leq x < n + 1.$$

Determine whether f is continuous.

3. The *(Cartesian) Product* of two sets X and Y is the set

$$X \times Y = \{(x, y) | x \in X \text{ and } y \in Y\}$$

If X and Y are both topological spaces, then we define the *product topology* on $X \times Y$ as the topology generated by the basis

$$B = \{U \times V | U \text{ is open in } X \text{ and } V \text{ is open in } Y\}$$

Define two maps

$$p_1 : X \times Y \to X \text{ and } p_2 : X \times Y \to Y$$

by $p_1(x, y) = x$ and $p_2(x, y) = y$. Prove that p_1 and p_2 are continuous.

4. Let X, Y, and Z be topological spaces and let $f : Z \to X \times Y$. Then f specifies two *coordinate functions* $f_1 : Z \to X$ and $f_2 : Z \to Y$ in the sense that we can think of $f(z) = (f_1(z), f_2(z))$.

(a) Prove that f is continuous if and only if both of its coordinate functions f_1 and f_2 are continuous.
(b) Use part (a) to prove that the *diagonal map* $\Delta : X \to X \times X$ given by $\Delta(x) = (x, x)$ is continuous for any topological space X.