MAT 300
Spring 2019
Homework # 2
Due: Friday, 2/1/19

Directions: Neatly write solutions to each of the following problems on separate sheets of paper. (You can put multiple problems on one page). Staple this page to the front of your finished proofs. Minimal to no credit will be given for solutions without appropriate justification. Each problem is worth 5 points.

1. Let $X = \{a, b, c, d\}$. Determine which of the following lists of sets form a topology on X.

   (1) $\emptyset, \{a\}, \{a, b\}, \{a, b, c, d\}$

   (2) $\emptyset, \{a\}, \{b\}, \{a, b, c, d\}$

   (3) $\emptyset, \{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, c, d\}$

   (4) $\{a\}, \{a, b\}, \{a, b, c\}, \{a, b, c, d\}$

2. Verify that the open sets $B_\delta(x, y) = \left\{ (x', y') : \sqrt{(x' - x)^2 + (y' - y)^2} < \delta \right\}$ do form a topology on $\mathbb{R}^2$.

3. Let $M : \mathbb{R}^2 \to \mathbb{R}$ be the multiplication function $M(x, y) = xy$. Sketch the preimage of the interval $(1, 2)$ and prove that this preimage is open in $\mathbb{R}^2$.

4. Prove that a function $f : S \to T$ between two topological spaces is continuous if and only if $f^{-1}(U)$ is closed in S whenever U is closed in T.