

MAT 300  
Spring 2019  
Homework # 1  
Due: Friday, 1/25/19

*Directions:* Neatly write solutions to each of the following problems on separate sheets of paper. (You can put multiple problems on one page). Staple this page to the front of your finished proofs. Minimal to no credit will be given for solutions without appropriate justification. Each problem is worth 5 points.

1. Prove that the interval  $[-1, 1]$  is closed by proving that its complement in  $\mathbb{R}$  is open.
2. Prove that the subset  $\mathbb{Z} \subset \mathbb{R}$  is closed.
3. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = x^3 - 3x$ . Calculate the preimages of the sets  $[-2, 2]$ ,  $(2, 18)$ ,  $[2, 18)$ , and  $[0, 2]$ .
4. Use the  $\epsilon - \delta$  definition of continuity to prove that each of the following functions are continuous.
  - $f(x) = x$
  - $f(x) = 3 - 2x$
5. Let  $\{A_i\}_{i \in I}$  be a collection of subsets of a given set  $X$ . (The indexing set  $I$  may be uncountable). Prove “DeMorgan’s Laws”:
  - (a)  $X - \bigcup_{i \in I} A_i = \bigcap_{i \in I} (X - A_i)$
  - (b)  $X - \bigcap_{i \in I} A_i = \bigcup_{i \in I} (X - A_i)$