MAT 147
Spring 2018
Homework

The following list of problems will continue to grow throughout the semester. You should check it frequently for updates. These problems will never be collected, but your quizzes and exams will be based on what appears below. Please ask questions about the problems below early and often!

1. Write the following sets using the “roster method”. That is, write the sets in list form.

(a) $A = \{x \in \mathbb{N} : -13 \leq x \leq 5\}$
(b) $B = \{x \in \mathbb{N} : x$ appears in the decimal expansion of $375/999\}$
(c) $C = \{x : x$ is the name in English of a month of the year$\}$
(d) $D = \{x : x$ is a prime number divisible by $2\}$
(e) $E = \{x : x$ is an integer less than $1\}$

2. List the next three elements in each of the following sets.

(a) $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots\right\}$
(b) $\{1, 2, 3, 5, 8, 13, \ldots\}$
(c) $\{-1, 3, -9, 27, \ldots\}$

3. Given the following sets, answer either true or false to each of the statements (a) - (h).

$$A = \{4, 8, 12, \ldots, 96, 100\} \qquad B = \{-1, 0, 1, 2, 3, 4, 5, 6\} \qquad C = \emptyset$$

$$D = (-\infty, -7] \qquad E = [-1, 6] \qquad F = (-1, \infty)$$

(a) $-7 \in D$
(b) $B \subset E$
(c) $0 \in A$
(d) $-1 \in F$
(e) $0 \in C$
(f) $C \subset A$
(g) $\{8\} \in A$
(h) $E \subset F$

4. The **power set** of a given set A is the set of all subsets of A, and is denoted by $\mathscr{P}(A)$. Find the set $\mathscr{P}(\{\varnothing, \{\varnothing\}\})$.

5. Which of the following are propositions?
(a) All men are mortal.
(b) What a surprise!
(c) The millionth digit in the decimal expansion of $\sqrt{3}$ is 5.
(d) Every goople is an aardling.
(e) The moon is made of green cheese.

6. Let P and Q be propositions. Make a truth table to show that the propositions $\sim (P \wedge Q)$ and $\sim P \vee \sim Q$ are equivalent. (That is, verify that their columns in the truth table are identical).

7. "NOR" circuits are commonly used as a basis for flash memory chips. If P and Q are propositions, then P NOR Q is defined to be the negation of $P \vee Q$. Make a truth table for P NOR Q.

8. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{1, 3, 5, 7\}$, $C = \{0, 1, 3, 4, 6, 7\}$, and $D = \{0, 1, 2, 4, 5, 6, 7, 8, 9\}$. Find each of the following:

(a) $A \cup B$

(b) $A - B$

(c) $(A - B) - C$

(d) $(A \cap C) \cap D$

(e) $(B \cap C) \cup (C \cap D)$

9. Use the Euclidean algorithm to compute each of the following:

(a) $gcd(6643, 2873)$

(b) $gcd(513, 187)$

(c) $gcd(12091, 8439)$

10. In each of the following cases, the missing number in the congruence is one of the integers $0, 1, 2$, or 3. Determine which one of these integers makes the congruence true, and be sure to show your work or give an explanation.

(a) $26 \equiv ?\ (mod\ 4)$

(b) $-17 \equiv ?\ (mod\ 4)$

(c) $99^4 \equiv ?\ (mod\ 4)$

11. Make an addition table for $\mathbb{Z}_6 = \{[0]_6, [1]_6, [2]_6, [3]_6, [4]_6, [5]_6, \}$

12. Find the prime factorization of each of the following integers: 10, 24, 513, 720, and 2520.

13. For each $n = 1, 2, \ldots, 11$, determine which elements of $\mathbb{Z}_n$ are divisors of zero and which elements of $\mathbb{Z}_n$ are units.

14. Verify that $\mathbb{Z}_8^\times$ is closed under the operation of multiplication. That is, make a multiplication table for $\mathbb{Z}_8^\times$ to show that all of the entries in the table are again elements of $\mathbb{Z}_8^\times$.

15. (a) Convert each of the following decimal numbers to binary notation: 19, 41, 137

(b) Convert each of the binary numbers from part (a) to hexadecimal notation.

(c) Use "long addition" to add the hexadecimal numbers that you found in part (b). Then convert your answer back to decimal notation..

(d) Reconcile your answer for part (c) with the numbers from part (a).

16. Verify the following string of equalities by hand.

$$719 + 25 = 222122_3 + 221_3 = 1000120_3.$$

17. Verify each of the following by hand:

(a) $123456 = 1E240_{16}$

(b) $110010111011_2 = CBB_{16}$

(c) $11010111010111110111101_2 = 6BAFBD_{16}$

18. Use Fermat's Little Theorem to solve $a^{35} \equiv 4 \pmod 5$.

19. Use the method of successive squares to compute $3^{550} \pmod{151}$.