

MAT 147
Phi Function Project
Due: Thursday, October 26th, 2017

Directions: Write careful solutions to each of the steps below separate sheets of paper. (You may put more than one solution on the same sheet of paper, if you have enough room). Be sure to show all of your work. You are allowed to talk to your classmates about these problems. If you do receive help from a classmate, be sure to give them credit by noting their name on your solution. All solutions should be written in your own words, regardless of if you've received help. Partial credit is available. This project is worth 5% of your total grade

1. Compute $\phi(n)$ for $n = 1, \dots, 20$.
2. Find a pattern for $\phi(n)$ when n is prime.
3. Can you find a pattern for $\phi(n)$ when n is a composite?
4. Compute $\phi(2^k)$ for $k = 1, \dots, 5$. Then repeat these 5 computations for $\phi(3^k)$, $\phi(5^k)$, $\phi(7^k)$, and $\phi(11^k)$. Can you see a general formula for $\phi(p^k)$ where p is a prime number and k is any power of p ?
5. Verify that $\phi(4 \cdot 3) = \phi(4) \cdot \phi(3)$, $\phi(25 \cdot 2) = \phi(25) \cdot \phi(2)$, and $\phi(9 \cdot 8) = \phi(9) \cdot \phi(8)$. Then verify that $\phi(3 \cdot 6) \neq \phi(3) \cdot \phi(6)$ and $\phi(2 \cdot 4) \neq \phi(2) \cdot \phi(4)$. Can you figure out, in general, when $\phi(mn) = \phi(m) \cdot \phi(n)$?
6. Find a general formula for $\phi(n)$ for any integer $n > 1$. (Recall that we can write $n = p_1^{k_1} p_2^{k_2} \cdots p_j^{k_j}$. Apply the phi function to both sides of this equation and, using what we've found above, try to "break down" $\phi(p_1^{k_1} p_2^{k_2} \cdots p_j^{k_j})$ as far as you can!)