

MAT 302
Assignment 3
Monday, February 18, 2013

For full credit on these problems, each must be submitted with a complete and clear solution, showing all of your work. You may work with other classmates on these problems, but please indicate on your assignment if you received help. Partial answers and incomplete solutions may be eligible for some partial credit, depending on the level of completeness and demonstrated understanding.

1. For the function $f(x, y) = x^2 - xy + y^3$, and the direction vector $\vec{u} = \left(\frac{5}{13}, \frac{12}{13}\right)$, find $D_{\vec{u}}f(x, y)$ using
 - (a) the definition of the directional derivative.
 - (b) the gradient of $f(x, y)$.
2. Find the direction of maximum increase of the function $f(x, y) = x^3 \sin y - x \cos y$ at the point $(5, \pi/3)$.
3. The temperature on a metal plate is given by

$$T(x, y) = 400 - 2x^2 - y^2.$$

Find the path of a heat-seeking particle placed at the point $(10, 10)$.

4. Show that the surfaces defined by the equations

$$x^2 + y^2 + z^2 - 8x - 12y + 4z + 42 = 0,$$

and

$$x^2 + y^2 + 2z = 7,$$

are tangent to each other at the point $(2, 3, -3)$ by finding the tangent planes to each surface at each point, and showing that they are the same.

5. Consider the elliptic cone given by

$$x^2 - y^2 + z^2 = 0.$$

- (a) Find an equation of the tangent plane at the point $(5, 13, -12)$.
- (b) Find symmetric equations of the normal line at the point $(5, 13, -12)$.