# MAT 334 Quiz 2

February 11, 2005

1. Find an example of each of the following. *For each of the following, the answers may vary. But I will give a couple of examples for each one.*

   (a) An infinite group.
   $\mathbb{Z}$ under addition, $\mathbb{Q}$ under addition.

   (b) A group of order 5
   $\mathbb{Z}_5$

   (c) An Abelian group
   $\mathbb{Z}, \mathbb{Z}_4, \mathbb{Z}_{10}$, etc.

   (d) A non-Abelian group
   $D_n$ for any $n > 3$.

   (e) A non-cylic group
   $D_n$ for any $n > 3$.

2. Find the orders of all elements of $\mathbb{Z}_{10}$.

   The orders of the elements $1, 3, 7$ and $9$ are all 10, since they all generate $\mathbb{Z}_{10}$.
   The orders of the elements $2, 4, 6$, and $8$ are all 5, since they all generate $\{0, 2, 4, 6, 8\}$.
   The order of the element 0 is 1, since it generates $\{0\}$.
   The order of the element 5 is 2, since it generates $\{0, 5\}$.

3. Find the orders of all elements of $U(10)$.

   The order of the element 1 is 1, since it generates $\{1\}$.
   The order of the elements 3 and 7 is 4, since they generate $\{1, 3, 7, 9\}$.
   The order of the element 9 is 2, since it generates $\{1, 9\}$.

4. List all the generators of the cyclic group $\mathbb{Z}_{20}$.

$$1, 3, 7, 9, 11, 13, 17 \text{ and } 19$$

5. The following Cayley table is the Cayley table of the group $S_3$. Answer the following questions about the group $S_3$.

| | (1) | (123) | (132) | (12) | (13) | (23) |
|---|---|---|---|---|---|---|
| (1) | (1) | (123) | (132) | (12) | (13) | (23) |
| (123) | (123) | (132) | (1) | (13) | (23) | (12) |
| (132) | (132) | (1) | (123) | (23) | (12) | (13) |
| (12) | (12) | (23) | (13) | (1) | (132) | (123) |
| (13) | (13) | (12) | (23) | (123) | (1) | (132) |
| (23) | (23) | (13) | (12) | (132) | (123) | (1) |

(a) What is the order of the group $S_3$?

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(b) Is the group $S_3$ Abelian?

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(c) What is the center $Z(S_3)$?

$\{(1)\}$

(d) What is the centralizer of the element (12) in $S_3$?

$\{(1), (12)\}$

(e) What is the centralizer of the element (123) in $S_3$?

$\{(1), (123), (132)\}$