

MAT 305

Assignment 10

Thursday, November 15, 2012

For full credit on these problems, each must be submitted with a complete and clear solution, showing all of your work. You may work with other classmates on these problems, but please indicate on your assignment if you received help. Partial answers and incomplete solutions may be eligible for some partial credit, depending on the level of completeness and demonstrated understanding.

1. The group of all symmetries of a square is called D_4 , the dihedral group. If you label the vertices of the square 1 through 4, you can consider each symmetry as a permutation of $A = \{1, 2, 3, 4\}$.
 - (a) How many elements does D_4 have?
 - (b) Write out all elements of D_4 as permutations of A , and choosing effective notation for the permutations.
 - (c) Make a group table for D_4 for the functional composition operation on D_4 .
 - (d) Make a Cayley diagram for the group D_4 using one reflection and one rotation as generators.
 - (e) Is this group commutative? That is, does $a \circ b = b \circ a$ for all $a, b \in D_4$?
 - (f) Is this group cyclic? That is, is there a particular symmetry that generates the entire dihedral group?
 - (g) Is this group “the same” as the group of symmetries of an 8 spoke pinwheel? Why or why not?