5. Identify all suspicious points and determine all points of discontinuity for

\[ f(x) = x^3 - 7x + 3 \]

Since \( f \) is a polynomial, there is nothing suspicious going on, and hence \( f \) is continuous everywhere.

6. Identify all suspicious points and determine all points of discontinuity for

\[ f(x) = \frac{3x + 5}{2x - 1} \]

When we divide, we must be careful not to divide by zero. So we should be suspicious when the denominator equals 0. That is, when

\[ 2x - 1 = 0. \]

When we solve for \( x \), we get

\[ x = \frac{1}{2} \]

So \( x = \frac{1}{2} \) is a suspicious point. To determine if this is a point of discontinuity, we plug this value into the function. But we determined that 1/2 is not in the domain of \( f \). That is, \( f(1/2) \) does not exist. So \( x = 1/2 \) is a point of discontinuity.

12. Identify all suspicious points and determine all points of discontinuity for

\[ g(t) = \begin{cases} 
3t + 2 & \text{if } t \leq 1 \\
5 & \text{if } 1 < t \leq 3 \\
3t^2 - 1 & \text{if } t > 3 
\end{cases} \]

I get suspicious when a function changes the rule for its’ definition. For the piecewise defined function \( g \), this occurs when \( t = 1 \) and \( t = 3 \). (At \( t = 1 \), the rule changes from \( g(t) = 3t + 2 \) to \( g(t) = 5 \), and at \( t = 3 \), the rule changes from \( g(t) = 5 \) to \( g(t) = 3t^2 - 1 \). So the suspicious points are \( t = 1 \) and \( t = 3 \).

However, these need not be points of discontinuity. Recall the definition (when in doubt, always refer to the definition) of continuity. We say a function \( f \) is continuous at \( x = a \) if the following three conditions hold.

1. \( f(a) \) exists
2. \( \lim_{x \to a} f(x) \) exists
3. \( \lim_{x \to a} f(x) = f(a) \).
We need to check these three conditions for each suspicious point \( t = 1 \) and \( t = 3 \). First we will investigate \( t = 1 \).

1. Since \( g(1) = 3(1) + 2 = 5 \), \( g(1) \) indeed exists.

2. This is the part that is a little hard. To determine if the limit exists, both the left-hand limit and the right-hand limit must exist and equal the same value. So we first need to compute \( \lim_{t \to 1^-} g(t) \). As we approach \textit{from the left}, we are using the expression \( 3t + 2 \), since numbers to the left of 1 are smaller than 1. So

\[
\lim_{t \to 1^-} g(t) = \lim_{t \to 1^-} 3t + 2 = 3(1) + 2 = 5
\]

We also need to compute \( \lim_{t \to 1^+} g(t) \). As we approach \textit{from the right}, we are using the expression 5, since numbers to the right of 1 are larger than 1. So

\[
\lim_{t \to 1^+} g(t) = \lim_{t \to 1^+} 5 = 5
\]

Since these two limits are equal, \( \lim_{t \to 1^-} g(t) \) exists and equals 5.

3. This part is easy. Since we got 5 for both the first two parts, we have \( \lim_{t \to 1^-} g(t) = g(1) \).

So, \( g \) passed all three criteria for being continuous at \( t = 1 \). Hence, \( g \) is continuous at \( t = 1 \). We must also investigate \( t = 3 \).

1. \( g(3) = 5 \) (so it exists)

2. We follow the same reasoning as the first part. Here we must look at the left-hand and right-hand limits. To the left of 3, we use the rule \( g(t) = 5 \), and to the right of 3, we use the rule \( g(t) = 3t^2 - 1 \). So,

\[
\lim_{t \to 3^-} g(t) = \lim_{t \to 3^-} 5 = 5
\]

and

\[
\lim_{t \to 3^+} g(t) = \lim_{t \to 3^+} 3t^2 - 1 = 3(3)^2 - 1 = 26
\]

Since these two limits are different, \( \lim_{t \to 3^-} g(t) \) does not exist! There is no need to check the third criterion. If it fails to satisfy one, it fails.

So \( g \) is not continuous at \( t = 3 \). To summarize: The function \( g \) has suspicious points at \( t = 1 \) and \( t = 3 \), and \( g \) is discontinuous at \( t = 3 \).

20. \textit{Find the value that should be assigned to} \( f(2) \), \textit{if any, to guarantee that} \( f \) \textit{will be continuous at} \( x = 2 \), \textit{for the function}

\[
f(x) = \frac{1}{\frac{x - 1}{x - 2}}
\]
Note that out of the three criteria for continuity at a point \( x = a \), two of them require something from \( f(a) \). The other one requires something out of the limit as \( x \) approaches \( a \). For this problem, \( a = 2 \). So let’s find \( \lim_{x \to 2} f(x) \). By cavalierly (not naively!) plugging in \( x = 2 \), we see that we get

\[
f(2) = \frac{\frac{1}{2} - 1}{2 - 2} = \frac{-\frac{3}{2}}{0},
\]

which is a nice reasonable number \(-3/2\) divided by 0. So this limit does not exist! So there can be no value that we can assign to \( f(2) \) to make this function continuous. In particular, this function has a pole at \( x = 2 \). That is, the graph will have a vertical asymptote at \( x = 2 \).

22. Determine whether or not the given function is continuous on the prescribed interval, where

\[
f(x) = \begin{cases} 
  x^2 & \text{if } 0 \leq x < 2 \\
  3x + 1 & \text{if } 2 \leq x < 5
\end{cases}
\]

Here the prescribed interval is \([0, 5]\), and the only place we should be suspicious of is \( x = 2 \), since the function changes its definition there. To the left of 2, \( f(x) = x^2 \), and to the right of 2, \( f(x) = 3x + 1 \). To be continuous at \( x = 2 \), we require the three conditions to hold.

1. \( f(2) = 3(2) + 1 = 7 \) (which exists)

2. We check the left-hand and right-hand limits.

\[
\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = 2^2 = 4
\]

and

\[
\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3x + 1 = 3(2) + 1 = 7.
\]

Since these two values are different, \( \lim_{x \to 2} f(x) \) does not exist, and so the function \( f \) is not continuous at \( x = 2 \).

32. Show that the given equation has at least one solution on the indicated interval.

\[\cos(x) = x^2 - 1 \text{ on } (0, \pi)\]

Let us consider the continuous function

\[f(x) = \cos(x) - x^2 + 1\]

on this interval. At the endpoints, we have

\[f(0) = \cos(0) - 0^2 + 1 = 1 - 0 + 1 = 2\]

which is positive, and

\[f(\pi) = \cos(\pi) - (\pi)^2 + 1 = -1 - \pi^2 + 1 = -\pi^2\]
which is negative. So by the Intermediate Value Theorem, there exists some $c \in (0, \pi)$ such that $f(c) = 0$. Or in other words, There is a $c \in (0, \pi)$ such that

$$\cos(c) - c^2 + 1 = 0,$$

or

$$\cos(c) = c^2 - 1.$$