MAT 201
Assignment 4
Tuesday, March 3, 2015

For full credit on these problems, each must be submitted with a complete and clear solution, showing all of your work. You may work with other classmates on these problems, but please indicate on your assignment if you received help. Partial answers and incomplete solutions may be eligible for some partial credit, depending on the level of completeness and demonstrated understanding.

1. For the following problems, find the indicated limit, if it exists. If it does not exist, explain why.

   (a) \( \lim_{x \to 3^+} \sqrt{x^2 - 9} \)
   (b) \( \lim_{x \to 0^+} \ln x \)
   (c) \( \lim_{x \to 2^+} \frac{|x - 2|}{x - 2} \)
   (d) \( \lim_{x \to 2^-} \frac{|x - 2|}{x - 2} \)

2. Find the values of the constants \( a, b, \) and \( c \), to make the following function continuous on the entire real number line, and sketch a graph of the function \( f(x) \) that you obtain.

   \[
   f(x) = \begin{cases} 
   4, & x \leq -3 \\
   ax + b, & -3 < x < -1 \\
   2x + c, & -1 \leq x < 4 \\
   5, & x \geq 4 
   \end{cases}
   \]

3. Find all points of discontinuity for the function \( f(x) = \csc (4x - 1) \).

4. Is the following statement true or false? Explain your answer.

   If \( f(x) = g(x) \) for all \( x \neq 2 \) and \( f(2) \neq g(2) \), then either \( f(x) \) or \( g(x) \) is not continuous at \( x = 2 \).

5. Consider the function \( f(x) = x + \sin x - 1 \). Explain why there must exist a real number \( a \) such that \( f(a) = 0 \). Give an estimate of a value \( a \) such that \( f(a) = 0 \) that is accurate to within 0.01.