MAT 301
Assignment 1
Monday, February 4, 2013

For full credit on these problems, each must be submitted with a complete and clear solution, showing all of your work. You may work with other classmates on these problems, but please indicate on your assignment if you received help. Partial answers and incomplete solutions may be eligible for some partial credit, depending on the level of completeness and demonstrated understanding.

1. Describe the domain and range of the function
   \[ f(x, y) = \sqrt{36 - 4x^2 - 9y^2}, \]
   and draw the level curves for all integer values of \( C \) that the function equals.

2. According to the Ideal Gas Law, \( PV = kT \) where \( P \) is pressure, \( V \) is volume, \( T \) is temperature (in Kelvins), and \( k \) is a constant of proportionality. A tank contains 2000 cubic inches of nitrogen at a pressure of 26 pounds per square inch and a temperature of 300 K.
   (a) Determine \( k \)
   (b) Write \( P \) as a function of \( V \) and \( T \) and describe the level curves.

3. Use the definition of the limit of a function of two variables to prove that
   \[ \lim_{{(x,y) \to (a,b)}} y = b \]

4. For each of the following, find the limit or explain why it does not exist.
   (a) \( \lim_{{(x,y) \to (0,0)}} \frac{xy}{x^2+y^2} \)
   (b) \( \lim_{{(x,y) \to (2,2)}} \frac{x^2-y^2}{x-y} \)

5. If \( f(1,3) = 2 \), can you conclude \( \lim_{{(x,y) \to (1,3)}} f(x,y) = 2 \)? Explain.

6. If \( \lim_{{(x,y) \to (1,3)}} f(x,y) = 2 \), can you conclude \( f(1,3) = 2 \)? Explain.