MAT 334 Quiz 1

January 28, 2005

1. Fill in the Cayley table for the group of symmetries of the equilateral triangle.

	R_0	R_{120}	R_{240}	μ_1	μ_2	μ_3
R_0	R_0	R_{120}	R_{240}	μ_1	μ_2	μ_3
R_{120}	R_{120}	R_{240}	R_0	μ_2	μ_3	μ_1
R_{240}	R_{240}	R_0	R_{120}	μ_3	μ_1	μ_2
μ_1	μ_1	μ_3	μ_2	R_0	R_{240}	R_{120}
μ_2	μ_2	μ_1	μ_3	R_{120}	R_0	R_{240}
μ_3	μ_3	μ_2	μ_1	R_{240}	R_{120}	R_0

2. Is the group of symmetries of the equilateral triangle Abelian? If not, why?

   No. To be Abelian means that $ab = ba$ for ALL a and b in the group. So to show that a group is not Abelian, we only need to find one pair of elements that do not commute. By looking at the table, we see that

$$\mu_1\mu_2 = R_{240}$$

   and

$$\mu_2\mu_1 = R_{120}$$

   Since $R_{120} \neq R_{240}$, this group is not Abelian.

3. Is the set $\mathbb{Z}$ under addition a group? If not, why?

   Yes it is. An identity element for $\mathbb{Z}$ is 0, since $0 + a = a$ for any a. The inverse of any element is its' negative. For instance, the inverse of 4 is -4, since $4 + (-4) = 0$. That is, the inverse of a is $-a$, since $a + (-a) = 0$ for any integer a. And addition is associative, which is cool.

4. Is the set $\mathbb{Z}$ under multiplication a group? If not, why?

   No it is not. To be a group, every element needs to have an inverse. Consider the number 2. Which *integer* can we multiply by 2 to a get 1? Give up? So do I. Thus, 2 does not have an inverse, and so $\mathbb{Z}$ is not a group under multiplication.