

## MAT 201 Quiz 2

February 11, 2005

1. Consider the function

$$f(x) = \frac{3x-1}{x+2}$$

- (a) What is the domain of  $f$ ?

$$\mathbb{R} \setminus \{-2\}$$

- (b) Evaluate  $f(2)$

$$f(2) = \frac{3(2)-1}{2+2} = \frac{5}{4}$$

2. Consider the functions  $f(x) = 2x + 1$  and  $g(x) = x^2$ .

- (a) Find the composition  $f \circ g$ .

$$f \circ g(x) = f(g(x)) = f(x^2) = 2x^2 + 1$$

- (b) Find the composition  $g \circ f$ .

$$g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2$$

- (c) Find the inverse  $f^{-1}(x)$ .

To find the inverse of a function  $f$ , substitute  $y$  for  $f(x)$  and switch the  $x$ 's and  $y$ 's. So we have

$$x = 2y + 1$$

When we solve for  $y$  we get

$$y = \frac{x-1}{2}$$

and so

$$f^{-1}(x) = \frac{x-1}{2}$$

- (d) Evaluate the difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(2(x+h) + 1) - (2x + 1)}{h} \\ &= \frac{2x + 2h + 1 - 2x - 1}{h} \\ &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

3. Simplify

$$\tan(\sin^{-1}(x))$$

If we let  $y = \sin^{-1}(x)$  then we have  $\sin(y) = x$ . So we are asked to simplify

$$\tan(y)$$

where  $\sin(y) = x$ . If we make a reference triangle, the opposite side will be  $x$ , and the hypotenuse will be 1. This forces the adjacent side to be  $\sqrt{1-x^2}$  to satisfy the pythagorean identity. Hence

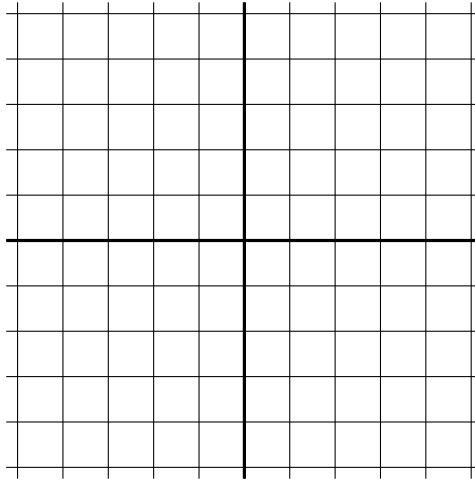
$$\tan(y) = \frac{\text{opposite}}{\text{adjacent}} = \frac{x}{\sqrt{1-x^2}}$$

4. Evaluate

$$\sin^{-1}\left(-\frac{1}{2}\right).$$

$$-\frac{\pi}{6}$$

5. Consider the graph of the function  $f$  given below:



- (a) Find  $\lim_{x \rightarrow -2^+} f(x)$ . 3
- (b) Find  $\lim_{x \rightarrow -2^-} f(x)$ . 1
- (c) Find  $\lim_{x \rightarrow -2} f(x)$ . D.N.E. since the above limits are different.
- (d) Find  $\lim_{x \rightarrow 3^-} f(x)$ . -2
- (e) Find  $\lim_{x \rightarrow 3^+} f(x)$ . -3
- (f) Find  $f(3)$ . -2