Problem 30 from Section 4.4

April 26, 2005

30.

$$f(x) = \frac{2x^2 - 5x + 7}{x^2 - 9}$$

The vertical asymptotes will occur where the denominator equals 0. This occurs when $x^2 - 9 = 0$ or when $x = 1$ and $x = -1$. The horizontal asymptotes will occur where the function has a limiting value in the long run. That is, what is

$$\lim_{x \to \infty} f(x)$$

and

$$\lim_{x \to -\infty} f(x)$$

In both cases, the limit is 2, so there will be a horizontal asymptote at $y = 2$.

Now we investigate the derivative

$$f'(x) = \frac{5(x-9)(x-1)}{(x^2-9)^2}$$

The critical numbers occur where the derivative equals 0 or does not exist. In this case, the critical numbers are

$$x = 1, 9, 3 \text{ and } -3$$

These 4 numbers split the real number line into 5 intervals

$$(-\infty, -3), (-3, 1), (1, 3), (3, 9), (9, \infty)$$

By testing the first derivative on each interval we see that the function is

$$\begin{array}{rll} \text{Increasing} & \text{on} & (-\infty, -3) \cup (-3, 1) \cup (9, \infty) \\ \text{Decreasing} & \text{on} & (1, 3) \cup (3, 9) \end{array}$$

Now we investigate the second derivative

$$f''(x) = \frac{-10(x^3 - 15x^2 + 27x - 45}{(x^2-9)^3}$$

which has second order critical points at

$$x = 3, -3 \text{ and } x \approx 13.2$$

By testing the second derivative on the appropriate intervals we find the function is

$$\begin{array}{rll} \text{Concave Up} & \text{on} & (-\infty, -3) \cup (3, 13, 2) \\ \text{Concave Down} & \text{on} & (3, 3) \cup (13.2, \infty) \end{array}$$

To summarize :

Vertical Asymptote	at	$x = 3$ and $x = -3$
Horizontal Asymptote	at	$y = 0$
Critical Points	at	$x = 1, 9, 3, -3$
Increasing	on	$(-\infty, -3) \cup (-3, 1) \cup (9, \infty)$
Decreasing	on	$(1, 3) \cup (3, 9)$
Second Order Critical Points	at	$x = 3, -3, 13.2$
Concave Up	on	$(-\infty, -3) \cup (3, 13, 2)$
Concave Down	on	$(3, 3) \cup (13.2, \infty)$

For the graph of this function, follow the link from the page labelled number 30.