1. Let $f(n) = 1 + 2 + 2^2 + 2^3 + \cdots + 2^n$.
   (a) Find $f(1), f(2), f(3)$ and $f(4)$.
   (b) Make a conjecture about the value of $f(n)$ for arbitrary $n$.
   (c) Use the Principle of Mathematical Induction to prove your claim in part (b).

2. Use the Principle of Mathematical Induction to prove that for all $n \in \mathbb{N}$, $8 \mid 9^n - 1$.

3. Use the differentiation formulas $\frac{d}{dx}(x) = 1$ and $\frac{d}{dx}(fg) = f \frac{dg}{dx} + g \frac{df}{dx}$ to prove that for all $n \in \mathbb{N}$, $\frac{d}{dx}(x^n) = nx^{n-1}$.

4. Use the Principle of Complete Induction to prove the following statement. Let 
   \[ \alpha = \frac{1 + \sqrt{5}}{2}, \quad \text{and} \quad \beta = \frac{1 - \sqrt{5}}{2}, \]
   be the solutions to the quadratic equation $x^2 = x + 1$. Let $f_n$ denote the $n$th Fibonacci number. Prove that 
   \[ f_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}. \]