MAT 301
Assignment 4
Friday September 21, 2012

For full credit on these problems, each must be submitted with a complete and clear solution, showing all of your work. You may work with other classmates on these problems, but please indicate on your assignment if you received help. Partial answers and incomplete solutions may be eligible for some partial credit, depending on the level of completeness and demonstrated understanding.

1. Using Taylor’s Theorem and the fourth Taylor polynomial for \( f(x) = e^x \), centered at \( c = 0 \), find an approximation to \( e^2 \). Use Taylor’s Theorem to find a bound for the error of the approximation. How do your answers compare to the value of \( e^2 \) that your calculator gives you?

2. When an elementary function \( f(x) \) is approximated by a second-degree polynomial \( P_2 \) centered at \( c \), what is known about \( f \) and \( P_2 \) at \( c \)? Explain your reasoning.

3. Find the interval of convergence for the following power series
   
   (a) \( \sum_{n=0}^{\infty} \left( \frac{x-2}{5} \right)^n \)
   
   (b) \( \sum_{n=0}^{\infty} \frac{(-1)^n(x+1)^n}{2^n} \)

4. Consider the function \( f(x) = \frac{3}{2x-5} \).

   (a) Find a power series for the function \( f(x) \) centered at \( c = 0 \).
   
   (b) Find the radius of convergence of the power series you found in part (a).
   
   (c) Find a power series for the function \( f(x) \) centered at \( c = 1 \).
   
   (d) Find the radius of convergence of the power series you found in part (b).
   
   (e) What is the domain of the function \( f(x) \). How does this make sense with the answers you found in the previous parts of the problem?