

MAT 300
Assignment II
Tuesday, September 4, 2007

You should attempt all the problems, and hopefully get complete solutions to a couple of them. Some of the problems are a little tricky, so don't give up if you do not get the correct solution right way. Keep working! The only way to get better at problem solving is to practice problem solving.

1. In how many ways can 10 be expressed as a sum of 5 *nonnegative* integers, when order is taken into account?
2. Given n objects arranged in a row. A subset of these objects is called *unfriendly* if no two of its elements are consecutive. Show that the number of unfriendly subsets each having k elements is $\binom{n-k+1}{k}$.
3. Let $a(n)$ be the number of representations of the positive integer n as a sum of 1's and 2's taking order into account. Let $b(n)$ be the number of representations of n as a sum of integers greater than 1, again taking order into account and counting the summand n .
 - (a) Show that $a(n) = b(n+2)$ for each n , by describing a one-to-one correspondence between the a -sums and b -sums.
 - (b) Show that $a(1) = 1$, $a(2) = 2$, and for $n > 2$, $a(n) = a(n-1) + a(n-2)$.