MAT 300
Assignment I
Tuesday, August 28, 2007

You should attempt all the problems, and hopefully get complete solutions to a couple of them. Some of the problems are a little tricky, so don't give up if you do not get the correct solution right way. Keep working! The only way to get better at problem solving is to practice problem solving.

1. Let S_1 denote the sequence of positive integers $1, 2, 3, 4, 5, 6, \ldots$ and define the sequence S_{n+1} in terms of S_n by adding 1 to those integers in S_n which are divisible by n. Thus, for example, S_2 is $2, 3, 4, 5, 6, 7, \ldots$ S_3 is $3, 3, 5, 5, 7, 7, \ldots$. Determine those integers n with the property that the first $n-1$ integers in S_n are n.

2. Prove that a list can be made of all the subsets of a finite set in such a way that

   (a) the empty set is first in the list,

   (b) each subset occurs exactly once, and

   (c) each subset in the list is obtained either by adding one element to the preceding subset or by deleting one element of the preceding subset.

3. Two poles, with heights a and b, are a distance d apart (along level ground). A guy wire stretches from the top of each of them to some point P on the ground between them. Where should P be located to minimize the total length of the wire?

4. A rectangular room measures 30 feet in length and 12 feet in height, and the ends are 12 feet in width. A fly, with a broken wing, rests at a point one foot down from the ceiling at the middle of one end. A smudge of food is located one foot up from the floor at the middle of the other end. The fly has just enough energy to *walk* 40 feet. Show that there is a path along which the fly can walk that will enable it to get to the food.

5. Let a and b be given positive real numbers with $a < b$. If two points are selected at random from a straight line segment of length b, what is the probability that the distance between them is at least a? (Hint : Let x and y denote the randomly chosen numbers from the interval $[0, b]$, and consider these independent random variables on two separate axes. What area corresponds to $|x - y| \geq a$?)