

MAT 104 Quiz 16

Friday, October 22, 2004

1. Factor completely

$$49x^2 - 25y^2$$

This is the difference of two squares, which has a factorization as

$$a^2 - b^2 = (a + b)(a - b)$$

where $a = 7x$ and $b = 5y$. So

$$49x^2 - 25y^2 = (7x + 5y)(7x - 5y)$$

2. Factor

$$49x^2 - 70xy + 25y^2$$

When you encounter something like this, where the first and last terms are perfect squares, first try

$$49x^2 - 70xy + 25y^2 = (7x \pm 5y)(7x \pm 5y)$$

and check if this works for any combination of signs. If both signs are minus signs, it fits. So

$$49x^2 - 70xy + 25y^2 = (7x - 5y)(7x - 5y)$$

3. Factor

$$2x^3 - 54y^3$$

Remember, your first step in factoring is to pull out the gcd of all of the terms, which is 2. So

$$2x^3 - 54y^3 = 2(x^3 - 27y^3)$$

and the remaining polynomial is a difference of cubes. The difference of cubes has a factorization as

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

where for us $a = x$ and $b = 3y$. So

$$\begin{aligned} 2x^3 - 54y^3 &= 2(x^3 - 27y^3) \\ &= 2(a^3 - b^3) \\ &= 2(a - b)(a^2 + ab + b^2) \\ &= 2(x - 3y)(x^2 + x(3y) + (3y)^2) \\ &= 2(x - 3y)(x^2 + 3xy + 9y^2) \end{aligned}$$